

Gas-like state of α clusters around ^{16}O core in ^{24}Mg

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We have studied gas-like states of α clusters around an ^{16}O core in ^{24}Mg based on a microscopic α -cluster model. This study was performed by introducing a Monte Carlo technique for the description of the THSR (Tohsaki Horiuchi Schuck Röpke) wave function, and the coupling effect to other low-lying cluster states was taken into account. A large isoscalar monopole ($E0$) transition strength from the ground to the gas-like state is discussed. The gas-like state of two α clusters in ^{24}Mg around the ^{16}O core appears slightly below the 2α -threshold energy.

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I. INTRODUCTION

In the last couple of years, gas-like states comprised of α clusters in atomic nuclei have attracted increased interest [1–6]. It is basically considered that all of the α -clusters in such states occupy the same $0s$ -orbital, which has a spatially extended distribution [2]. This situation is well expressed by introducing the so-called THSR (Tohsaki Horiuchi Schuck Röpke) wave function, where the oscillator parameter is large, which is completely different from the normal $0s$ -orbital for each nucleon. The most plausible candidates for such gas-like states of α clusters are the second 0^+ state of $^{12}\text{C}^*$ (3α) at $E_x = 7.65$ MeV and more generally, states around the $N\alpha$ -threshold energies. The squared overlap between the wave function of a microscopic cluster model and the THSR wave function has been found to be more than 90% for ^{12}C [3, 4], which suggests that the single THSR wave function is a good approximation for the description of the gas-like state. Furthermore, a candidate for the gas-like state of 4α clusters around the threshold energy in ^{16}O has been studied by both theoretical and experimental approaches [1, 3, 7].

Lately, the research into gas-like states comprised of α clusters moved onto the second stage. For instance, the possibility of gas-like states of α clusters around a core in heavier nuclei has been suggested [8–11]. The coherent emission of α clusters from the compound nucleus in heavier nuclei has been reported [8], which leads us to a hypothesis that gas-like states of α clusters can be formed not only in ^{12}C and ^{16}O but also in heavier nuclei with some core. Using the $^{28}\text{Si}+^{24}\text{Mg}$ reaction, the compound states of ^{52}Fe have been populated, and the $^8\text{Be}(0_1^+)$ and $^{12}\text{C}(0_2^+)$ emissions from these states have been observed, which are much enhanced compared to the sequential α -emission. From a statistical model point of view, it is natural to consider that the emitted second 0^+ state of ^{12}C is formed inside the Coulomb barrier of the compound nucleus. The enhancement of the emission of the

gas-like states of α clusters could be due to the lowering of the effective Coulomb barrier for the condensed states [9], since the kinetic energy of the emitted ^{12}C in coincidence with γ -emission has been observed to be much smaller than the energy sum of three α 's in the sequential 3α emission.

To study the possibility of gas-like states of α clusters in heavier nuclei from the theoretical side, we have introduced a Monte Carlo technique for the description of the THSR wave function, which is called the “virtual THSR” wave function [12]. We have shown the possibility of a three α state around a ^{40}Ca core [13]. The three α cluster state around ^{40}Ca has an energy below the Coulomb barrier top energy with a spatial extension comparable to the second 0^+ state of ^{12}C .

Recently, it has been proposed that the strong enhancement of isoscalar monopole ($E0$) transitions can be a measure of the existence of cluster structure [14]. For instance, the presence of cluster states in ^{13}C has been suggested by measuring the isoscalar $E0$ transitions from the ground $1/2^-$ state induced by the $^{13}\text{C}(\alpha, \alpha')^{13}\text{C}$ reaction [15]. The obtained cross-sections are much larger than those of the shell-model calculations. The result suggests that protons and neutrons are coherently excited and that they have spatially extended distribution in the excited states. From the theoretical side, the relation between the monopole transition strength and the cluster structure has also been discussed [16–20]. The basic idea arises from the Bayman-Bohr theorem [21], which shows that the lowest representation of the shell-model contains a component of the lowest $SU(3)$ representation of the cluster states and the monopole operator connects the lowest $SU(3)$ state and the well-developed cluster states.

The experimental search for the large monopole strength to gas-like cluster state(s) of α clusters around an ^{16}O core is on going, and we analyze such structure from the theoretical point of view. In this paper we study the gas-like state of two α clusters around an ^{16}O core in ^{24}Mg , and here, the coupling effect between the gas-

like α cluster state and the normal cluster states in the low-lying region is taken into account.

II. METHOD

The original THSR wave function for the α -condensed state has the following form:

$$\begin{aligned}\Phi &= \int d\vec{R}_1 d\vec{R}_2 \cdots d\vec{R}_n \\ &\mathcal{A} G(\vec{r}_1, \vec{R}_1) G(\vec{r}_2, \vec{R}_2) G(\vec{r}_3, \vec{R}_3) \cdots G(\vec{r}_n, \vec{R}_n) \\ &\times \exp[-(\vec{R}_1^2 + \vec{R}_2^2 + \vec{R}_3^2 \cdots \vec{R}_n^2)/\sigma^2] \\ &= \mathcal{A} \prod_{i=1}^n \int d\vec{R}_i G(\vec{r}_i, \vec{R}_i) \exp[-\vec{R}_i^2/\sigma^2],\end{aligned}\quad (1)$$

where \mathcal{A} , $G(\vec{r}_i, \vec{R}_i) = \exp[-\nu(\vec{r}_i - \vec{R}_i)^2]$, and σ are the antisymmetrizer, the wave function for the i -th α -cluster centered at \vec{R}_i , and the oscillator parameter of the α -condensation, respectively. The four nucleons (proton spin-up, proton spin-down, neutron spin-up, and neutron spin-down) in the i -th α -cluster share the common Gaussian center parameter \vec{R}_i .

To simplify this wave function, we introduce the “virtual THSR” wave function in the following way [12]:

$$\Psi^\sigma = \sum_{k=1}^m P^\pi P_{MK}^J \Psi_k^\sigma, \quad (2)$$

$$\Psi_k^\sigma = [\mathcal{A} G(\vec{r}_1, \vec{R}_1) G(\vec{r}_2, \vec{R}_2) G(\vec{r}_3, \vec{R}_3) \cdots G(\vec{r}_n, \vec{R}_n)]_k. \quad (3)$$

Here, the integral over the Gaussian center parameters $\{\vec{R}_i\}$ in the original THSR wave function (in Eq. (1)) is replaced by the sum of many Slater determinants. The Gaussian center parameters ($\{\vec{R}_i\}$) are randomly generated by the weight function W with a Gaussian shape:

$$W(\vec{R}_i) \propto \exp[-\vec{R}_i^2/\sigma^2]. \quad (4)$$

With increasing ensemble number, the distribution of $\{\vec{R}_i\}$ approaches a Gaussian with σ width parameter. Thus, it can be considered that the integration in the original THSR wave function (see Eq. (1)) is performed by using a Monte Carlo technique for the virtual THSR wave function, and the wave function agrees with the original THSR wave function when the number of Slater determinants (m in Eq. (2)) increases.

In the present case, we add a ^{16}O core and discuss the two α state around it. Thus, each basis state (Eq. (3)) becomes

$$\Psi_k^\sigma = [\mathcal{A} \phi(^{16}\text{O}) G(\vec{r}_1, \vec{R}_1) G(\vec{r}_2, \vec{R}_2)]_k. \quad (5)$$

Here the ^{16}O core ($\phi(^{16}\text{O})$) is described by a tetrahedron configuration of four α -clusters with small relative distances, and only the Gaussian center parameters of two

α clusters around the ^{16}O core (\vec{R}_1, \vec{R}_2) are randomly generated with the weight function $W(\vec{R}_i)$. The center of mass of each basis state Ψ_k is shifted to the origin after generating the wave functions of α clusters around ^{16}O . The projection onto good parity (P^π) and angular momentum (P_{MK}^J) is performed numerically. Here, π is positive parity and $J = M = K = 0$. The number of mesh points for the Euler angle integral is $16^3 = 4096$. The Gaussian size of each nucleon (ν in $G(\vec{r}_i, \vec{R}_i) = \exp[-\nu(\vec{r}_i - \vec{R}_i)^2]$) is taken to be $\nu = 1/2b^2$, $b = 1.46$ fm.

In the present study, we calculate the coupling effect between the gas-like cluster states described by the virtual THSR wave function and normal cluster states. The normal cluster states $\{\Psi_{nc}^j\}$ are described by many different configurations of $^{16}\text{O} + \alpha + \alpha$ for ^{24}Mg . The total wave function Ψ is therefore

$$\Psi = \sum_j c_{nc}^j \Psi_{nc}^j + \sum_\sigma c^\sigma \Psi^\sigma, \quad (6)$$

and the coefficients $\{c_{nc}^j\}$ and $\{c^\sigma\}$ are determined by diagonalizing the Hamiltonian matrix.

The Hamiltonian operator (\hat{H}) has the following form:

$$\hat{H} = \sum_{i=1}^A \hat{t}_i - \hat{T}_{c.m.} + \sum_{i>j}^A \hat{v}_{ij}, \quad (7)$$

where \hat{t}_i is the kinetic energy of i -th nucleon, and the center-of-mass kinetic energy ($\hat{T}_{c.m.}$) is exactly removed. Here, the two-body interaction (\hat{v}_{ij}) includes the central part and the Coulomb part. We use the following Volkov No.2 effective $N - N$ potential [22]:

$$V(r) = (W - MP^\sigma P^\tau) \sum_{k=1,2} V_k \exp(-r^2/c_k^2), \quad (8)$$

where $W = 1 - M$ (M : Majorana exchange parameter). It is known that although $M \sim 0.6$ reproduces the α - α scattering phase shift, larger M values are needed for the structure calculation beyond ^{12}C , and here M is chosen to be 0.63 to yield a reasonable binding energy of ^{24}Mg . Using the present interaction, the lowest state of $^{16}\text{O} + 2\alpha$ (^{24}Mg) is calculated to be -16.24 MeV from the three-body threshold energy, when we perform a GCM (generator coordinate method) calculation by superposing the basis states and diagonalizing the Hamiltonian, compared with the experimental value of -14.046 MeV.

III. RESULTS

We start the discussion with the virtual THSR wave function for the gas-like state of α clusters around ^{16}O (Ψ^σ). The energy convergence of $^{16}\text{O} + \alpha + \alpha$ (^{24}Mg) as a function of number of basis states (m in Eq. (2)) is shown in Fig. 1. The energy is measured from the $^{16}\text{O} + \alpha + \alpha$ threshold. Our previous application of the virtual THSR

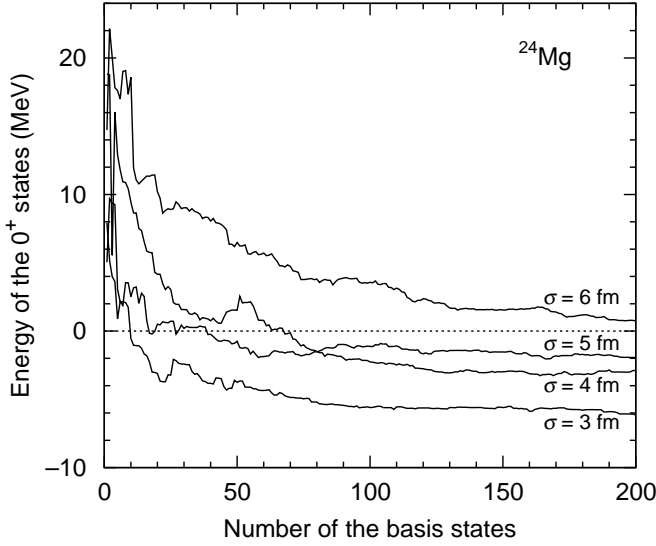


FIG. 1: The energy convergence of two α clusters around the ^{16}O core (^{24}Mg) for the case of $\sigma = 3, 4, 5$ and 6 fm. The energy is measured from the $^{16}\text{O}+2\alpha$ threshold.

wave function for the three α (^{12}C) case has shown that the wave function with $\sigma = 4$ fm is found to give a reasonable root mean square radius for the second 0^+ of ^{12}C compared with that obtained by other approaches [12]. In the present case, the result of $\sigma = 4$ fm (also that of $\sigma = 5$ fm) shows the converged energy below the threshold due to the presence of the ^{16}O core. In the case of $\sigma = 3$ fm, the converged energy is much lower; ~ -6 MeV below the threshold.

Next we calculate the coupling effect between the gas-like cluster states described by the virtual THSR wave function (Ψ^σ , $\sigma = 3, 4, 5$ and 6 fm) and normal cluster states (Ψ_{nc}^j). The left column of Fig. 2 shows the 0^+ levels of ^{24}Mg obtained by diagonalizing the Hamiltonian consisting of 200 different configurations of $^{16}\text{O}+\alpha+\alpha$. The energy convergence of $\{\Psi_{nc}^j\}$ is shown in Fig. 3. In addition to this normal cluster states of $\{\Psi_{nc}^j\}$, we superpose virtual THSR wave functions with the σ values of $\sigma = 3, 4, 5$ and 6 fm, and the result obtained after the coupling is shown as the right column of Fig. 2. In principle, the model space of Ψ_{nc} covers that of Ψ^σ if we prepare an infinite number of different configurations, however the energies of the calculated 7th 0^+ state at -4.267 MeV is influenced by the coupling with the virtual THSR wave function. This state is considered to have a large component of gas-like α clusters. Because of the presence of the ^{16}O core, the 7th 0^+ state appears below the threshold energy.

The energies of the eleven lowest 0^+ states of ^{24}Mg measured from the $^{16}\text{O}+\alpha+\alpha$ threshold (MeV) and squared isoscalar monopole (E0) transition strength from the ground state (fm^4) are summarized in Table I. The calculated monopole transitions from the ground state to the 7th 0^+ state at -4.267 MeV shows strong enhancement (187.5 fm^4). The result shows that the state which

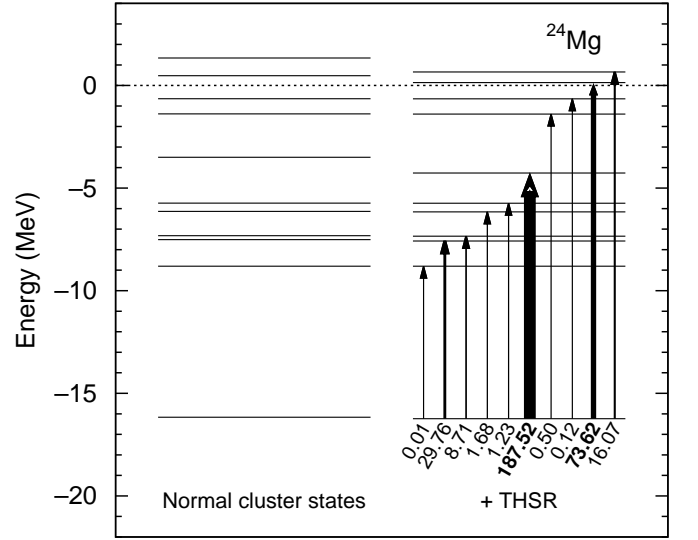


FIG. 2: The eleven lowest 0^+ states of ^{24}Mg obtained by the $^{16}\text{O}+\alpha+\alpha$ model. The left column shows the levels obtained by diagonalizing the Hamiltonian consisting of 200 different configurations of normal cluster states (Ψ_{nc}^j). The right column shows the results after superposing the virtual THSR wave functions (Ψ) with the σ values of $\sigma = 3, 4, 5$ and 6 fm in addition to Ψ_{nc}^j . The arrows and numbers (fm^4) show the E0 transition strength from the ground state.

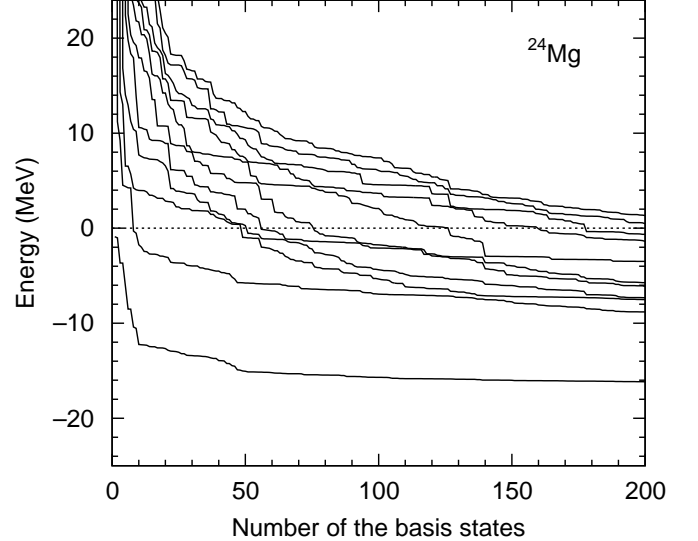


FIG. 3: The energy convergence of eleven lowest 0^+ states of ^{24}Mg obtained by the basis states of normal cluster states with the $^{16}\text{O}+\alpha+\alpha$ configurations (Ψ_{nc}^j).

has large components of gas-like cluster basis states is strongly excited by the E0 transition, which could be observed in experiments as a signature of well developed α cluster state.

The squared overlaps between each of the 0^+ states and the THSR wave functions for the two α clusters around ^{16}O ($\sigma = 3, 4, 5$, and 6 fm) are shown in Table II. The ground state has a large overlap with $\sigma = 3$ fm, because

TABLE I: The energies of the eleven lowest 0^+ states of ^{24}Mg measured from the $^{16}\text{O}+\alpha+\alpha$ threshold (MeV) and squared isoscalar monopole (E0) transition strength from the ground state (fm^4).

energy (MeV)	squared monopole strength (fm^4)
-16.24	—
-8.807	0.009
-7.580	29.76
-7.344	8.709
-6.157	1.684
-5.739	1.231
-4.267	187.5
-1.393	0.503
-0.650	0.118
0.140	73.62
0.658	16.07

TABLE II: The energies of the eleven lowest 0^+ states of ^{24}Mg measured from the $^{16}\text{O}+\alpha+\alpha$ threshold (MeV) and squared overlap with the THSR wave functions for the two α clusters around ^{16}O ($\sigma = 3, 4, 5$ and 6 fm).

energy (MeV)	$\sigma = 3$ fm	$\sigma = 4$ fm	$\sigma = 5$ fm	$\sigma = 6$ fm
-16.24	0.234	0.114	0.094	0.046
-8.807	0.000	0.000	0.000	0.000
-7.580	0.113	0.070	0.058	0.025
-7.344	0.023	0.014	0.012	0.005
-6.157	0.007	0.006	0.005	0.003
-5.739	0.003	0.002	0.002	0.001
-4.267	0.429	0.426	0.392	0.263
-1.393	0.001	0.002	0.002	0.002
-0.650	0.000	0.000	0.000	0.000
0.140	0.038	0.092	0.083	0.087
0.658	0.016	0.047	0.046	0.005

the limit of $\sigma = 0$ fm corresponds to the limit of the Elliot SU(3) state. It is confirmed that the 7th 0^+ state has a considerable amount of overlap with the THSR wave

functions with larger σ values (e.g. about 40% with $\sigma = 5$ fm). The 10th 0^+ state at 0.140 MeV just around the $^{16}\text{O}+\alpha+\alpha$ threshold energy, which also has a large E0 transition strength from the ground state (76.32 fm^4 in Table I), has a certain overlap with the THSR wave functions with large σ values.

IV. CONCLUSION

We have studied α cluster states around an ^{16}O core, which has been hinted at experimentally. The approach is based on a microscopic α cluster model. This was performed by introducing a Monte Carlo technique for the description of the THSR (Tohsaki Horiuchi Schuck Röpke) wave function, which is called the “virtual THSR” wave function.

The two α cluster state around an ^{16}O core in ^{24}Mg has an energy below the threshold energy, when the spatial extension is comparable to that of the second 0^+ state of ^{12}C . The character of this state survives after imposing the coupling with other cluster states in lower energy region. The 7th 0^+ state below the $^{16}\text{O}+\alpha+\alpha$ threshold has significant overlaps with the THSR wave functions for large σ values. The calculated isoscalar monopole (E0) transition from the ground state to this state shows strong enhancement, which could be observed as an experimental signature of a well developed α gas-like state.

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